



Department of Mathematics & Philosophy of Engineering

Faculty of Engineering Technology

The Open University of Sri Lanka

Course: MPZ 3132-Engineering Mathematics IB

Assignment No.03 Academic Year – 2011/2012

Instructions

- Answer all questions
- Write your address back of your answer scripts
- Use both sides of papers when you are doing assignment.
- Please send the answer scripts of your assignment **on or before the due date** to the following address.

Course Coordinator – MPZ 3132

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You can collect model answers from virtual class (www.ou.ac.lk)

User name - student0 Password – MPZ3132

1. Define the reduced row echelon form of a matrix.

1.1. Consider the following system of equations.

$$3x - 2y + z = 0$$

$$-11x + 8y + z = 0$$

$$10x - 7y + z = 0$$

$$-4x + 3y + z = 0$$

1.1.1. Express the above system of the form $AX = \mathbf{0}$ where $X = [x \ y \ z]^T$ and

$\mathbf{0} = [0 \ 0 \ 0]^T$. A is to be determined.

1.1.2. By applying elementary row operations find $RREM(A)$.

1.1.3. Deduce the solutions of the system of equations.

1.2. Using elementary row operations of matrices and augmented matrix solve the following system of equations for the x, y, z, t .

$$x + 2y + z + t = 1$$

$$3x + 3z - 2t = 11$$

$$x - y + z = 8.$$

2. Define the rank of a matrix.

2.1. Using the definition find the rank of the following matrix.

2.2.
$$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 3 & 6 & 5 \\ 1 & -1 & -2 & 0 \end{bmatrix}$$

2.3. Using the elementary row operations find the RREM of the following matrix.

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 4 & 3 & 2 & 1 \\ 6 & 7 & 0 & 1 \\ 7 & 9 & -1 & 1 \end{bmatrix}.$$

2.4. Discuss the solutions of the following systems of linear equations and when the systems have solutions find the solutions.

2.4.1.
$$\begin{aligned} x + 2y &= \alpha \\ 3x + y &= \beta \\ 2x - y &= \gamma \quad \text{Where } \alpha, \beta, \gamma \in R \end{aligned}$$

2.4.2.
$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (\mu^2 - 14)z &= \mu + 2 \quad \text{Where } \mu \in R \end{aligned}$$

3. Define the argument of a complex number.

3.1. Shade the region satisfying the following both inequalities

3.1.1. $|z - 4| \leq 5$ and $2 \leq \text{Im}(z) \leq 3$.

3.1.2. $5 \leq |z - 3| \leq 16$ and $|z - 3| \leq |z - 4i|$.

3.1.3. $4 \leq |z - 5| \leq 9$ and $\frac{\pi}{6} \leq \arg(z) \leq \frac{2\pi}{3}$

3.2. Find the values of the following

3.2.1. $\text{Log}(4 + 3i)$

3.2.2. $(1 + i)^i$

3.3. Show that the function $f(z) = \frac{3z+4}{z-1}$ where $z \neq 1$ is one to one function and find its inverse function.

3.4. If $f(z) = z^2$, $|z| = 3$ and $0 \leq \arg(z) \leq \frac{\pi}{3}$ find the image and draw the domain and the image of the function f .

4. Define the binomial distribution.

4.1. If $X \sim \text{Bin}(n, p)$ prove that the moment generating function of X is $(pe^t + 1 - p)^n$

4.1.1. Hence prove that the mean and variance of X are np and $np(1 - p)$ respectively.

4.2. A sweet company sells fruit toffees in packets; each packet contains the same number of toffees. The number of mango toffees per packet has a binomial distribution with mean 11 and variance $4 \cdot 95$.

4.2.1. Find the number of toffees in a packet.

4.2.2. Find the probability that a packet contains exactly 8 number of mango toffees.

4.3. Another sweet company sells fruit toffees in packets of 100. This company says that the probability any mango toffee in a packet is $0 \cdot 3$. Using the normal approximation find the probability that a packet contains fewer than 35 mango toffees.

5. Prove that the moment generating function of the normal distribution which has the probability density function $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ where $x \in R$ is $M(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$. Hence find the mean and variance of the normal distribution.

5.1. The times taken to germinate for a certain kind of paddy seeds are normally distributed. If 20% of the seeds take more than six days to germinate and if 10% of the seeds germinate in less than 4 days, then find the mean and the standard deviation of the time for germination of the paddy seeds.

5.2. The height of a large batch of a certain crop are normally distributed with a mean of 140 cm and a standard deviation of 6cm.

5.2.1. Find the probability that the height of a randomly selected tree of that batch is less than 145 cm .

5.2.2. If five trees are selected at random from the above batch ,find the probability that all the five trees are less than 145 cm tall and only three of them are less than 145 cm .

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